COMPARING TRAJECTORIES BASED ON GPS ALTITUDES AND BARO CORRECTED ALTITUDES.

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Abstract

Recently a new method was developed for combining vertical acceleration data with height data (say by GPS or Barometric type instrument) and vertical velocity data to perform a calibration of the vertical acceleration data and thereby strongly enhance the accuracy of the calculated velocities and heights of an airplane during flight. During descend of the Tu-154M at Smolensk on the 10th of April 2010 the TAWS flight recorder logged a number of parameters at a number of events separated in time and space. Among the recorded data were heights measured by 3 different systems: GPS heights, radio heights and barometric heights. The radio and barometric heights seem to agree with each other within reasonable measurement uncertainties, but the GPS heights measured by the three independent GPS units in general measured significantly higher heights (about 60m higher) than the radio/baro heights. With other words the TAWS system recorded two sets of height data, GPS heights and Radio/Baro heights. ***In the work presented in this paper the method of combining the data from the distance, velocity and acceleration domains is used to determine which set of height data is most probable and in best agreement with the recorded vertical acceleration and recorded vertical sink rate data.*** *This is done by including the data of the TAWS 38. This is new compared to the paper describing the development of the method, where the TAWS 38 data merely served as a verification of the method itself. Two Monte Carlo simulations are performed one for each set of heights only changing the input height data from one set to the other. The results clearly show that the GPS height data set is in agreement with the laws of physics and the Radio/baro height data set are not and very unlikely to agree with the official explanation of the plane's left wing striking a birch tree at 5m above the ground. This result is furthermore supported by the fact, that the trajectory of the official explanation would require vertical accelerations far beyond the ability of the Tu-154M plane, whereas the trajectory calculated by the GPS data set is in full agreement with the normal performance of the Tu-154M.*

Keywords - GPS data, Wing Damage, Roll, Smolensk, TU-154, Monte Carlo Technique.

1. INTRODUCTION

The work presented here utilizes the small subset of data selected by the Russian and Polish authorities and published by the American company Universal Avionics. In particular the Baro heights, GPS heights and vertical velocities recorded at the TAWS 34 to 38 events are of interest in this work. The information of these data is combined with the vertical acceleration sensor data recorded by the Polish QAR data recorder on board by a newly developed method [1] taking all data from the three domains (distance, velocity and acceleration) into account. For unknown reasons this vertical acceleration data was cut at the point just prior to the second big dip (loss of lift). In this work an estimate of the average vertical acceleration the following 1 second is found based on the recorded vertical velocity at the TAWS 38. It is worth to note, that the final 1 second acceleration estimate has no effect of the trajectory prior this point itself, as it occurs at the end of the dataset rather than earlier.

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2. MODEL

The mathematical model used to combine the height data, vertical velocity data and vertical acceleration data is very simple and described in [1].

Tab. 1. The input data [1,2]. H_{26} is the GPS height above runway 26 assuming runway 26 has an altitude of 255m (MSL). H_{BC} is the barometric (corrected) height above the runway. Data at TAWS35* are interpolated between TAWS35 and TAWS36.

TAWS # [-]	TAWS Time [hr:m:s]	BARO H_{BC} [m]	GPS H_{26} [m]	Sink Vz [ft/min]	Sink Vz [m/s]
34	06:40:03	329.2	394.9	-1441	-7.32
35	06:40:29	168.6	231.3	-1336	-6.79
35*	06:40:30	160.1	222.0	-1364	-6.93
36	06:40:36	116.5	174.0	-1513	-7.69
37	06:40:43	61.1	130.4	-1505	-7.65
38	06:40:59	36.3	50.5	+394	+2.00

Tab. 2. The estimated uncertainty values and type of uncertainty distributions used in the Monte Carlo Simulation. The distribution of the uncertainty of the time of the individual TAWS events is assumed uniform, as the logging system rounds to an integer value. "D" denotes Diameter, "H" denotes Height, T denotes time and σ the standard deviation.

Parameter of Interest	Estimated Uncertainty	Type of Distribution
GPS Height	$D_{3\sigma}$ = 30 m	Gaussian
Vertical Speed	$H_{2\sigma}$ = ±35 ft/min	Gaussian
Time of Taws Event	ΔT = ±0.5 s	Uniform
Each Individual Vertical Acc. Data Point	σ = 0.01g	Gaussian
Scale Factor	σ = 0.01	Gaussian

3. Results

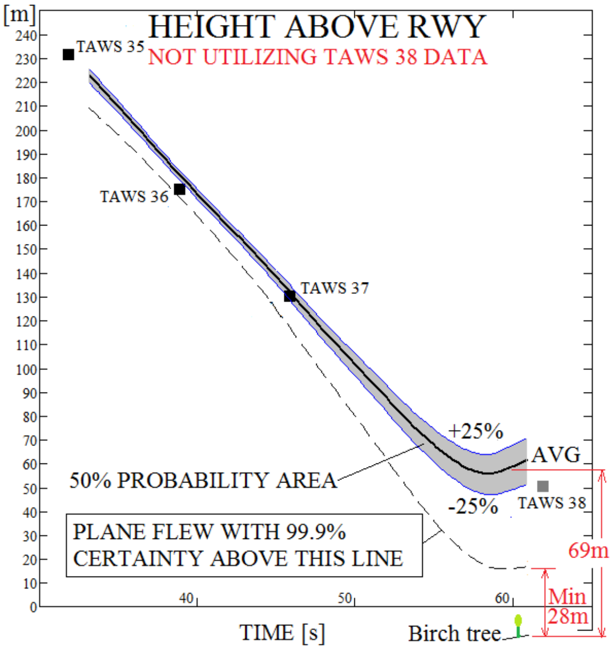

Figure 1. From [1] : Based on the recorded black box GPS heights, vertical velocities, times of logging and N=100.000 Monte Carlo simulations without including the TAWS 38 data the plane is with 99.9% certainty 28m above the ground at the vicinity of the Bodin Birch Tree, and most likely H = 69m ± 9m above this (Black Line) [1]. (The ground of the Birch Tree is 12m below the level of runway 26. [Error! Bookmark not defined.**])**

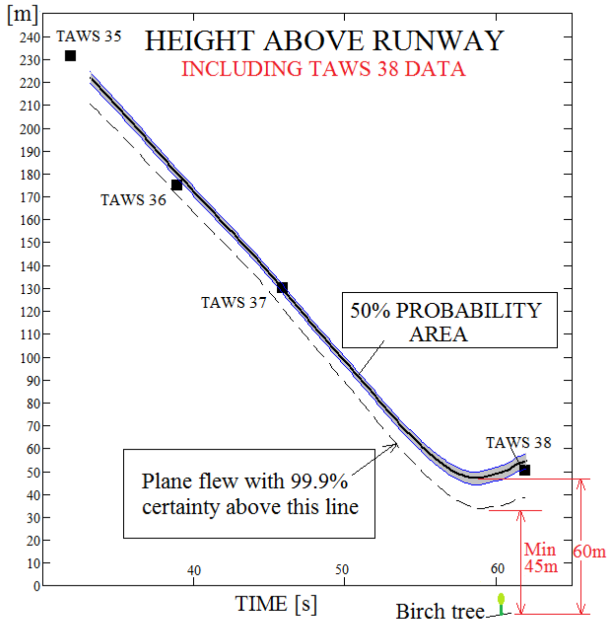

Figure 2. Based on the recorded black box GPS heights, vertical velocities, times of logging and N=15.000 Monte Carlo simulations including the TAWS 38 data the plane is with 99.9% certainty 45m above the ground at the vicinity of the Bodin Birch Tree, and most likely H = 60m ± 5m above this (Black Line). (The ground of the Birch Tree is 12m below the level of runway 26.)

3.1. Based on GPS Heights including TAWS 38 Data

3.1.1. Trajectory and Cumulative Probability Distribution.

The effect of measurement uncertainties of all the input parameters including data recorded at the TAWS 38 event is investigated using a Monte Carlo Simulation with the uncertainty estimates of each input parameter as listed in table 2 and N = 15.000 simulations finding the best least squared error fits as described above for each of these simulations. The results are shown in Figures 2. These results can be compared to the same when the recorded data at TAWS 38 is not included [1].

Including the recorded data at TAWS 38 removes additional uncertainty, as one can see by the corresponding "99.9% trajectory" and 50% probability area shown in figure 2. The most likely height above runway of the plane at the time the plane flew in the vicinity of the Bodin Birch tree when including the TAWS 38 data is H_{rwy}=48m and the plane is with 99.9% certainty H_{BT} = 45m above the ground near the birch tree.

Figure 3 shows the cumulative probability distributions of height above the vicinity of the ground near the Bodin birch tree for both cases including TAWS 38 data (red curve) and not including Taws 38 data (black curve). Note the narrowing of the distribution as one might expect when including the additional data set at TAWS 38.

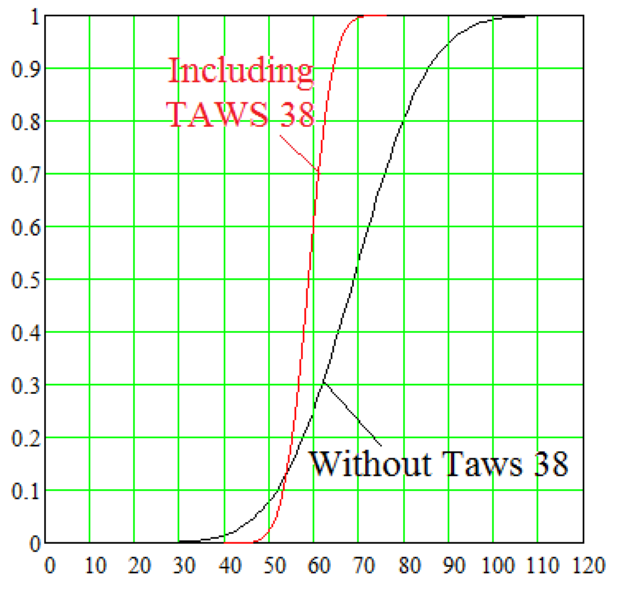

Figure 3. The cumulative probability distributions of height (H in m) above ground in the vicinity of the Bodin birch tree without including the TAWS38 data (black line) and including the TAWS 38 data (red line). Note the distribution becomes more narrow when including the TAWS 38 data, and the data show the plane with P=99.9% certainty was higher than 45.7m above the ground near the birch tree and most likely H = 60m ± 5m above the ground according to average and median of the Monte Carlo Simulations.

The 15.000 Monte Carlo simulations based on GPS height data taking uncertainties on the input parameters into account and including TAWS 38 data show:

1) The plane with 99.9% certainty flew more than 45.7m above the ground near the Bodin Birch tree and

2) The plane most likely flew 60m+/-5m above the ground near the Bodin Birch tree claimed by the Russians to have cut the wing. (This equals H_{rwy}=48m above the runway ground.)

3.1.2. Vertical Velocity based on GPS height data.

The calculated vertical velocity is found for the most likely trajectory shown in figure 2. The result is presented in

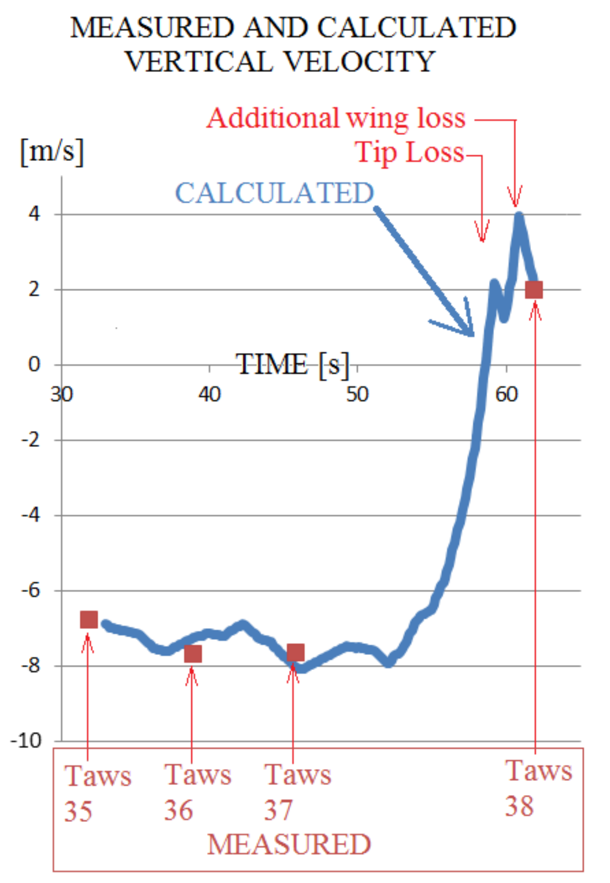

Figure 4. The calculated vertical velocities (blue points) and the recorded vertical velocities (red squares) at the taws 35-38 events for the most likely trajectory (see average curve of figure 2).

figure 4, and this shows a very good agreement between the calculated and measured vertical velocities.

The recorded sink rates correlate with the calculated sink rates for GPS height data.

3.2. Additional Results.

From figure 5 it can be seen, that the plane's height above the runway was close to H_{rwy}=100m, when the pilots stated they would initiate the "go-around" (abort the landing procedure). Thus the data clearly shows that the pilots actually did initiate the go-around at this moment. The calculated height loss during the go-around with an initial velocity of V_z=-7.7m/s is found by figure 5 to be ΔH = 45m. This agrees very well with the ΔH = 47m given in the Russian literature [2] for V_z=-7.7m/s (see figures 9 and 10).

The data show the pilots initiated the go-around within the second after they announced they would do this. This is in full agreement with what is normally expected from competent pilots, namely that they actually follow the command they loudly call in the cockpit, and as such not at all surprising.

The calculated height of the plane at the moment of the pilot's call of go-around is close to 100m above runway. 100m is the official decision height of this flight, i.e. the final point by which the pilots should make such decision of doing go-around or continuing the landing approach.

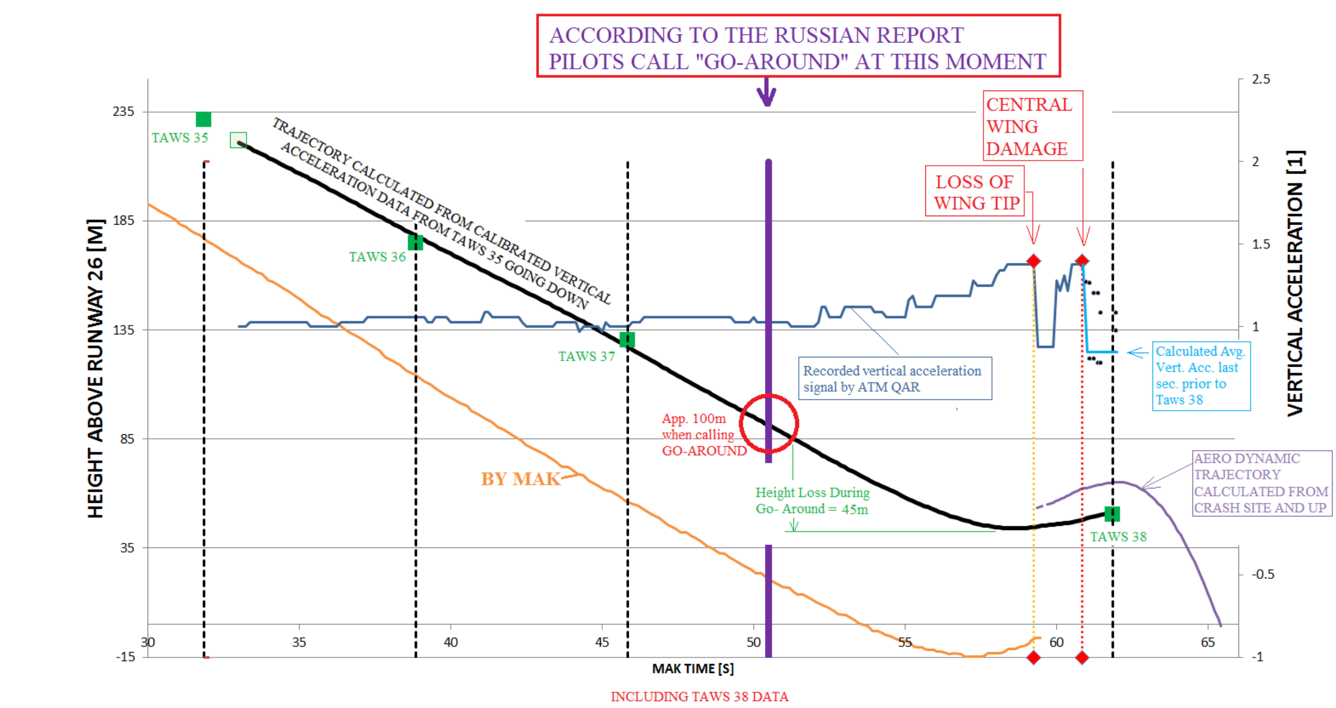

Figure 5. The plane was at about 100m height above the runway, at the time the pilots according to the official Russian report announced they would abort the landing procedure and initiate the go-around (see red circle). The data suggest, the pilots initiated the go-around within the second after their announcement on the radio. The calculated height loss of around 45m is in good agreement with the expected value for a TU-154M plane with the downwards vertical speed of about 7.7m/s at the moment the Go-Around was initiated.. The blue data are the raw data of the vertical acceleration sensor (ATM) (right side axis).

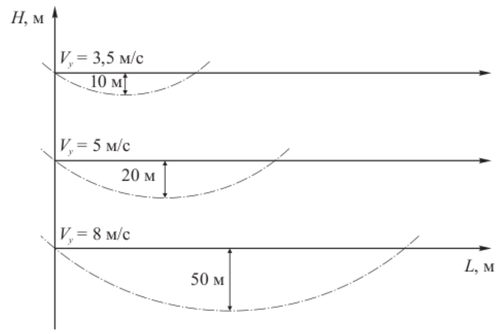

Рис. 6.10. Просадка самолета Ту-154М при уходе на второй круг

Figure 6. The estimated height loss, ΔH, of the airplane during a go-around maneuver for three different values of vertical velocities Vz (-3.5 m/s, -5m/s and -8 m/s). Based on this the height loss for an initial vertical velocity of Vz=-6.95m/s is ΔH = 39.5m (Russian source [6]).

The calculated height loss during the go-around agrees well with the expected height loss of the TU-154M as by the Russian literature confirming the results.

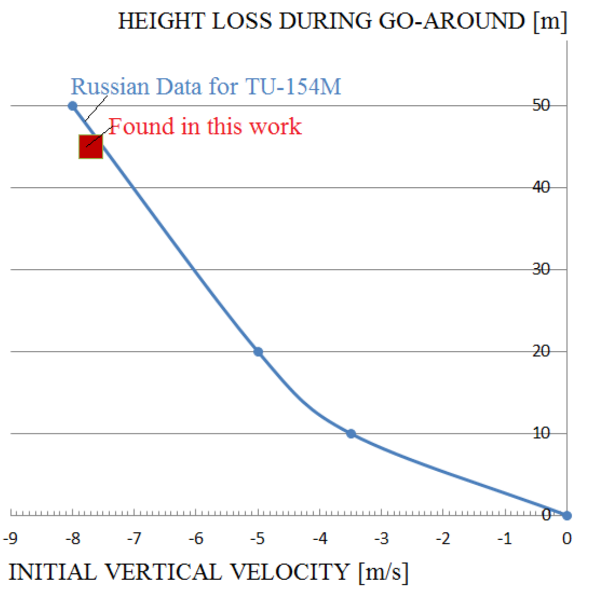

Figure 7. The estimated height loss, ΔH, of the airplane during a go-around maneuver for three different values of vertical velocities Vz (-3.5 m/s, -5m/s and -8 m/s). Based on this the height loss for an initial vertical velocity of Vz=-7.7m/s is ΔH = 47m (Source [6]).

3.3. Based on Baro Corrected Heights including TAWS 38 Data.

3.3.1. Trajectory and Cumulative Probability Distribution.

Figure 8 shows the cumulative probability distribution of the height above the ground near the Bodin birch tree by 15.000 Monte Carlo simulations using the baro corrected height data as input in comparison to the distribution obtained using GPS height data (both including TAWS 38 data). Figure 9 shows the trajectory plots for the case of using baro corrected height data as input. Note the correlation between the calculated trajectories and the recorded values are significantly weaker than the same for the case of using GPS height data (compare figure 2 and figure 9). By the Monte Carlo simulation based on the baro corrected heights the plane with P=99.9% certainty flew 18m above the ground near the birch tree area, and the median height above the ground by the barometric data is found to be 32m.

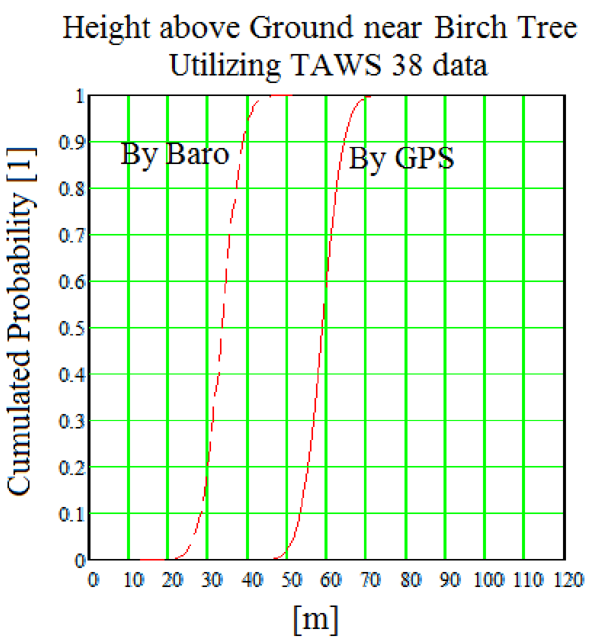

Figure 8. The cumulative probability distributions of height (H in m) above ground in the vicinity of the Bodin birch tree including the TAWS38 data based on barometric heights (dashed line) and GPS heights (full line). Note the distributions are offset by about 28m. By the simulations based on the barometric height data the plane with P=99.9% certainty was higher than 17m above the ground near the birch tree and most likely H = 33m ± 5m above the ground according to average and median of the Monte Carlo Simulations.

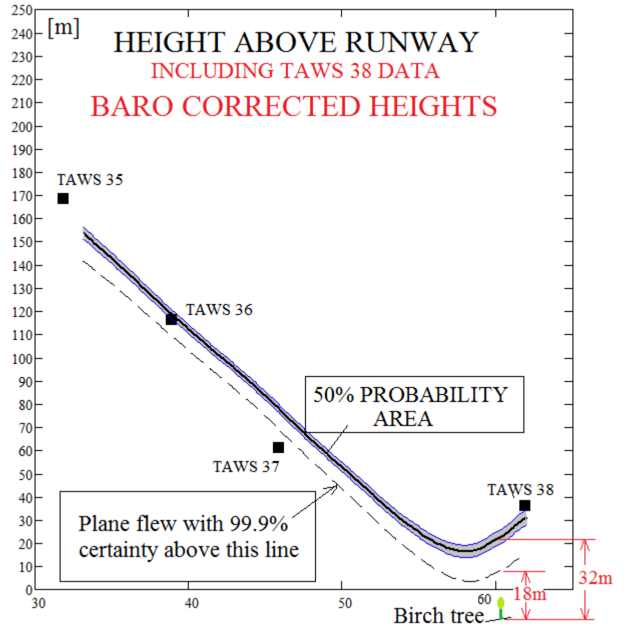

Figure 9. Based on the recorded black box Baro corrected heights, vertical velocities, times of logging and N=15.000 Monte Carlo simulations including the TAWS 38 data the plane is with 99.9% certainty 18m above the ground at the vicinity of the Bodin Birch Tree, and most likely H = 32m ± 5m above this (Black Line).

The 15.000 Monte Carlo simulations based on Baro corrected height data taking uncertainties on the input parameters into account and including TAWS 38 data show:

1) The plane with 99.9% certainty flew more than 18m above the ground near the Bodin Birch tree and

2) The plane most likely flew 32m+/-5m above the ground near the Bodin Birch tree claimed by the Russians to have cut the wing. (This equals H_{rwy}=20m above the runway ground.)

3.3.2. Vertical Velocity based on baro corrected height data.

The calculated vertical velocity is found for the most likely trajectory shown in figure 9. The result is presented in figure 10, and this shows a systematic disagreement agreement between the calculated and measured vertical velocities.

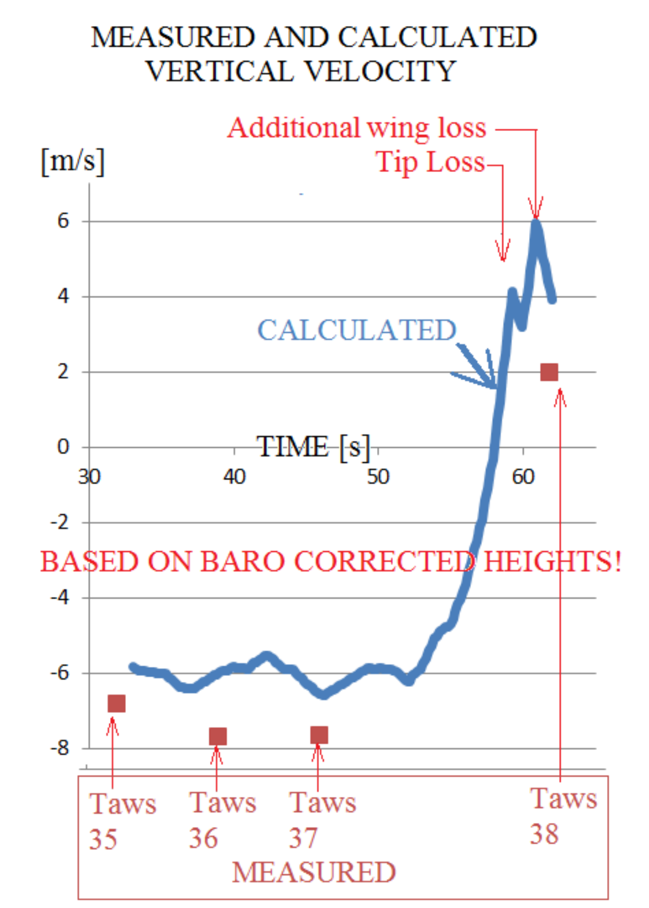

Figure 10. The calculated vertical velocities (blue points) using baro corrected height as input data and the recorded vertical velocities (red squares) at the taws 35-38 events for the most likely trajectory (see average curve of figure 9).

Based on baro corrected height data the calculated sink rates show a significant systematic error compared to the recorded sink rates.

4. Conclusion

The recorded GPS heights and baro/radio heights of the Tu-154M during the final descend do not match. The robust method of calibrating the vertical acceleration sensor data by utilizing the recorded GPS data and the recorded vertical velocity data at the TAWS 35 to TAWS 37 events [1] is used to determine which set of height data correlate best with the recorded sink rate data and vertical acceleration data.

- Both the calculated and recorded vertical sink rate data and the calculated and recorded height data clearly correlate best for the case of GPS height data.
- The recorded baro corrected heights are incompatible with the recorded sink rates and the recorded vertical acceleration data.
- The Monte Carlo simulation (N=15.000) including TAWS 38 data shows, that even by the baro corrected data the plane with P=99.9% certainty flew 18m above the ground in the vicinity of the Bodin birch tree claimed by the Russians to have cut the wing.

The Monte Carlo simulation (N=15.000) including TAWS 38 data and based on the GPS height data shows:

- That the plane with P=99.9% certainty flew 45.7m above the ground in the vicinity of the Bodin birch tree claimed by the Russians to have cut the wing.
- The plane most likely flew H_{rwy} = 60m±5m above the runway in the vicinity of the Bodin birch tree. This is within 10m of the calculated height at this point based on the aero dynamics working backwards from the crash site and up.[3]
- The height loss during go-around is found to be about 45m, which is in good agreement with the Russian data for the Tu-154M plane when the initial vertical velocity is V_z=-7.7m/s as found in this work.
- The pilots initiated the go-around at about H_{rwy} = 100m above the runway level within the second after the time they by the Russian transcript of the voice recorder claimed they would do go-around [4].

References

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